Sample Problems

Problem 1

What is the biggest number you can make by crossing out ***5*** digits from the ***10***-digit number ***2946835107***?

Hint Solution

Problem 2

There are two sums on the board

1 + 22 + 333 + 4444 + 55555 + 666666 +7777777 + 88888888 + 999999999
9 + 98 + 987 + 9876 + 98765 + 987654 + 9876543 + 98765432 + 987654321

Determine which one of them is bigger (or whether they are the same).

Hints Solution

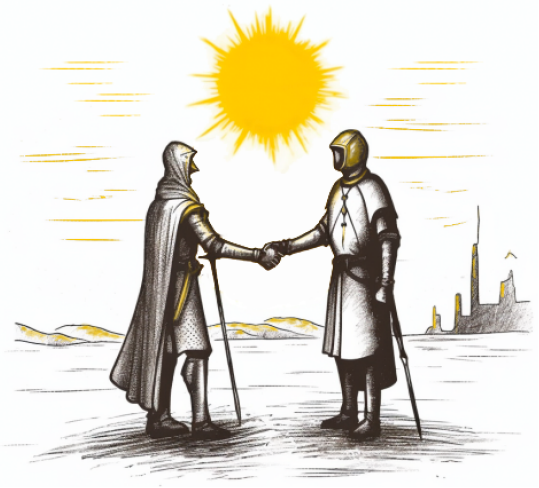
Problem 3

Three pirates found a treasure chest with ***240*** gold bars. These bars are worth a total of ***$360***. The price of each bar is an integer number of dollars, and it is written on the bar.

Could it be the case that the pirates cannot divide the treasure equally without melting or cutting any of the bars?

Hint Solution

Problem 4



Two knights, Lancelot and Galahad, started their journeys from castle ***A*** to castle ***B*** and from castle ***B*** to castle ***A***, respectively, at dawn at the exact same time.
Each walked at a steady pace for the whole trip. At noon, they met and said hello to each other. Then they continued on their journeys. Lancelot arrived at his destination at ***4*** pm, and Galahad arrived at his destination at ***9*** pm.

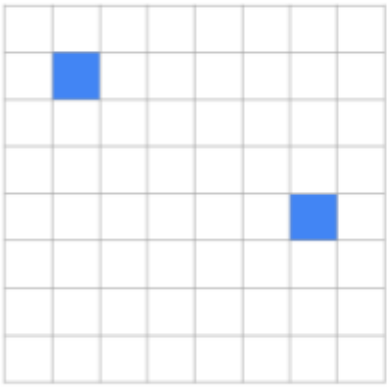
The question is: At what time did dawn occur on that day?

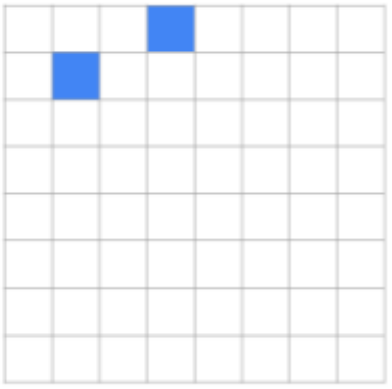
Hint Solution

Problem 5

In an ***8×8*** square, any two cells are colored in blue. Is it possible to divide the square into two equal (same size and same shape) parts so that each part contains one of the blue cells, wherever the blue cells are located?

Here are two examples of how blue cells can be placed in the square.





Hints Solution

HINTS to Problems

See the next pages.

Hint to Problem 1

To create the largest number, place the biggest digits in the leftmost positions.

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Hints to Problem 2

Hint 1

Write down each sum in column addition format.

Hint 2

What digits in the ones places do you need to add for each sum?
What digits in the tens places do you need to add for each sum?

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Hint to Problem 3

Yes, it could be the case.

You need to give an example and explain why the division is not possible in that example.

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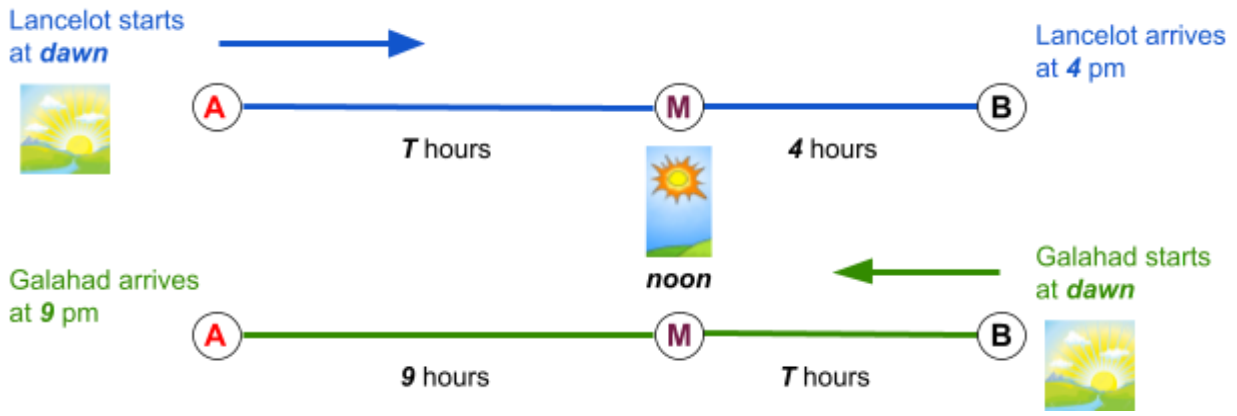
Hint to Problem 4



Denote by ***M*** the noon meeting point of Lancelot and Galahad and by ***T*** the time from the beginning of the journeys to the time of the meeting (from dawn to noon). Compare the ratio ***AM/MB*** and the time intervals traveled by each knight.

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Hints to Problem 5

Hint 1

Yes, it is always possible. You need to explain how to divide the square for ANY position of the blue cells.

Hint 2

Divide the original ***8x8*** square into ***4×4*** quarters. What would be your strategy if the blue cells were in different quarters? In the same quarter?

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Solutions to Problems

See the next pages.

Solution to Problem 1

The answer is ***98517***.

Indeed, the resulting number will have five digits. If we don't cross out the first digit in the number ***2946835107***, the resulting five-digit number will start with ***2***, making it smaller than ***98517***. Hence, we must cross out the first digit. We should also keep the digit ***9***, as any other digit would make the first digit of our five-digit number smaller than ***9***. By following this logic, we determine that the second digit should be the largest possible digit, so we cross out the digits ***4*** and ***6***, keeping the digit ***8*** as the second digit. Next, we need to cross out the digit ***3***, as keeping it would result in a five-digit number starting with ***983***, which is less than ***98517***. Now, only one digit remains to be crossed out, and it's easy to determine which one.

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Solution to Problem 2

Let's write down both sums in column form, with the second sum reversed for better visualization. In each sum, in the ones position, the summed digits are from ***1*** to ***9***; in the tens position, the digits are ***2*** to ***9***; in the hundreds, the digits are ***3*** to ***9***, etc. The digit obtained in each position and the carry-over into the next position to the left will be the same in both cases. Therefore, the results of the addition will be the same in either case.

$$
\begin{array}{r}
{\scriptstyle 4}\\
1\\
22\\
333\\
4444\\
+\quad 55555\\
666666\\
7777777\\
88888888\\
999999999\\
\hline
5
\end{array}
\qquad
\begin{array}{r}
{\scriptstyle 4}\\
987654321\\
+\quad 98765432\\
9876543\\
987654\\
98765\\
9876\\
987\\
98\\
9\\
\hline
5
\end{array}
$$

Both of these sums can be directly calculated, resulting in ***1097393685*** in both cases. However, this way of comparing them is overly complicated and boring.

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Solution to Problem 3

To divide the treasure equally, each pirate should get ***$120*** worth of gold. It is impossible if the price of ***1*** bar is ***$121*** and the price of the remaining ***239*** bars is ***$1*** each.

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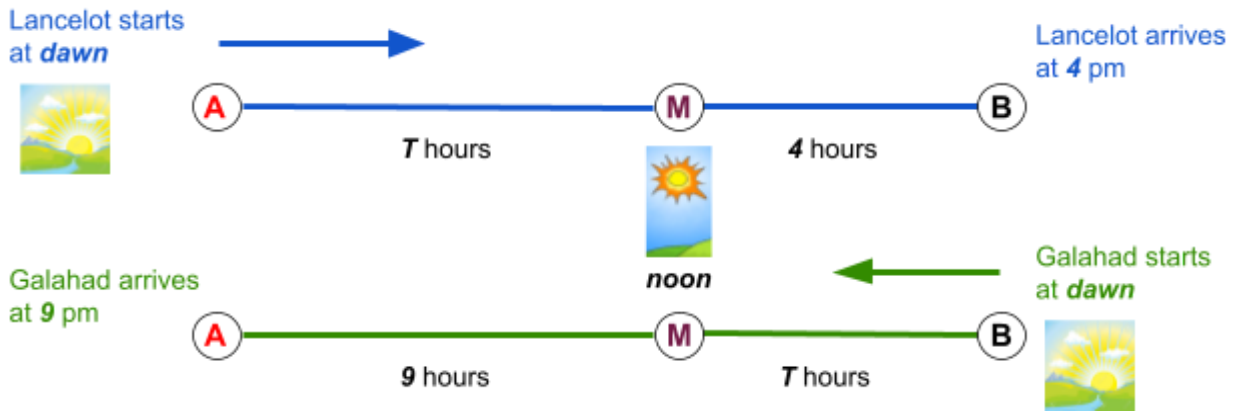
Solution to Problem 4



Let ***M*** be the noon meeting point of Lancelot and Galahad, and let ***T*** be the time from the beginning of the journeys to the time of the meeting (from dawn to noon).

The longer a knight walks, the bigger distance he covers.
Lancelot covers ***AM*** in ***T*** hours and ***MB*** in ***4*** hours, so the ratio ***AM/MB*** is equal to ***T/4***.
Similarly, Galahad covers ***AM*** in ***9*** hours and ***MB*** in ***T*** hours, so the ratio ***AM/MB*** is equal to ***9/T***.

Notice that the same ratio of ***AM/MB*** is expressed in two different ways, so ***9/T = T/4***.

Multiplying both sides by ***T*** and then by ***4***, we get $T^2 = 36$. Therefore, ***T = 6***, and dawn occurred ***6*** hours before noon or at ***6*** am.

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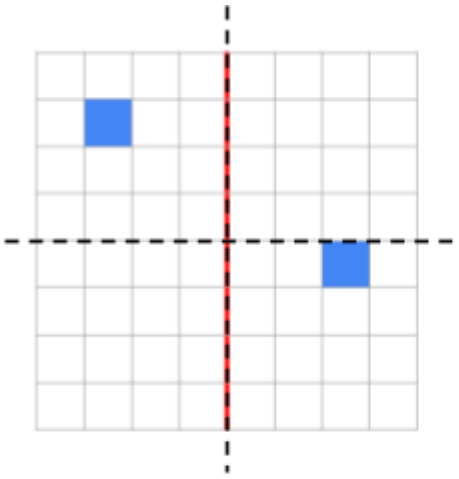
Solution to Problem 5

Yes, it is always possible.

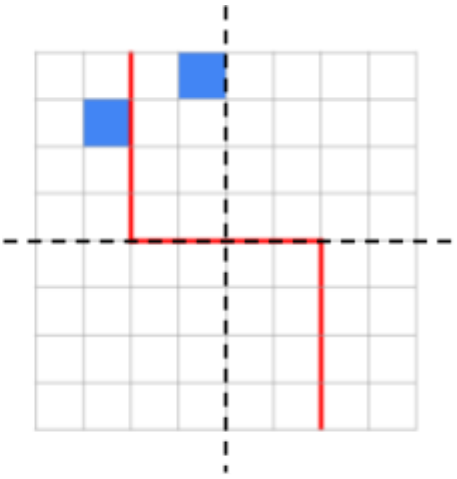
1. If the two blue cells are in different ***4×4*** quarters, you can cut along the horizontal or vertical middle line of the square. Picture 1 illustrates this case.

2. Let's examine the case when the two blue cells are in the same ***4×4*** quarter. We can rotate the square, placing the ***4×4*** quarter in the top left corner. The blue cells are either in different columns or in different rows (otherwise, these cells would coincide).

   Let’s assume that the blue cells are in different columns (similar logic applies if they were in different rows). Picture 2 illustrates this case. Let's cut the ***4×4*** quarter vertically so that the blue cells are in different parts of that quarter. Then we continue the cut horizontally along the border of the ***4×4*** quarter until we reach the center of the ***8×8*** square. Next, continue horizontally and count the same number of grid cells to the right of the center as we cut on the left. Finally, cut down the bottom right ***4×4*** quarter.



Picture 1



Picture 2

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